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Quantum Effects of a Nondissipative Mesoscopic Capacitance Coupling Circuit in a Displaced Squeezed Fock State

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Starting from the classical equation of motion for a mesoscopic capacitance coupling circuit, we study the quantum fluctuations of charge and current of the mesoscopic capacitance coupling circuit in a displaced squeezed Fock state. It is found that the quantum fluctuations of charge and current in each component circuit depend on the devices of two circuits and the squeezing parameters, while the fluctuations do not depend on displacement parameters.

1. INTRODUCTION

With the progress in nanotechnology and microelectronics, the trend in the miniaturization of integrated circuits and devices toward atomic-scale dimensions becomes stronger. When the transport dimension reaches a characteristic dimension, namely, the charge carrier inelastic coherence length, quantum effects must be taken into account. Louisell [1] first discussed the quantum effects of an *LC* circuit and gave its quantum noise in the vacuum state. This problem has become of interest due to the development of mesoscopic physics [2, 3] and its importance in future quantum computers [4]. Recently, the quantum fluctuations of a nondissipative mesoscopic capacitance coupling circuit in squeezed vacuum states have been investigated [5, 6]. In this paper, the quantum fluctuations of charge and current of a nondissipative mesoscopic capacitance coupling circuit in a displaced

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squeezed Fock state are investigated. As the displaced squeezed Fock state can be reduced to the vacuum state and squeezed vacuum state, the fluctuations of the mesoscopic capacitance coupling circuit discussed before are are special case of our general results. Therefore, it is more general and significant to study the quantum fluctuations of charge and current in displaced squeezed Fock states.

2. QUANTUM FLUCTUATIONS OF A MESOSCOPIC CAPACITANCE COUPLING CIRCUIT IN A DISPLACED SQUEEZED FOCK STATE

For two nondissipative LC circuits coupled via a capacitance in the presence of a source $\varepsilon(t)$ in one of two circuits (see Fig. 1), the classical equations of motion, as a consequence of Kirchhoff's law, read

$$L_1 \frac{d^2 q_1}{dt^2} + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C} = \varepsilon(t) \tag{1a}$$

$$L_2 \frac{d^2 q_2}{dt^2} + \frac{q_2}{C_2} - \frac{q_1 - q_2}{C} = 0 \tag{1b}$$

where $q_k(t)$ (henceforth we do not indicate the subscript $k = 1, 2$) are the charges of the two circuits, C_k and L_k stand for the capacitance and inductance of each component circuit, respectively, and C is the coupling capacitance between the two circuits. When $\varepsilon(t) = 0$, Eqs. (1) take the simple Hamilton form

$$H = \frac{p_1{}^2}{2L_1} + \frac{p_2{}^2}{2L_2} + \frac{q_1{}^2}{2C_1} + \frac{q_2{}^2}{2C_2} + \frac{(q_1 - q_2)^2}{2C} \tag{2}$$

where the variables q_k stand for the electric charges instead of the conventional "coordinates" and their conjugate variables $p_k = L_k\, dq_k/dt$ represent (apart

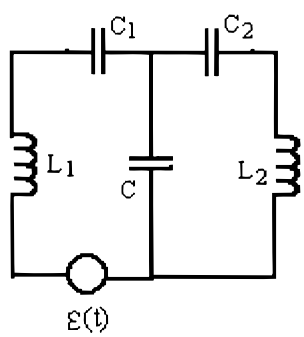

Fig. 1. Nondissipative capacitance coupling circuit.

from the factors L_k) the electric currents instead of the conventional "momenta." According to the standard quantization principle, one associates with each of two observable quantities q_k and p_k linear Hermitian operators, $\hat{q}_k$ and $\hat{p}_k$ satisfy the commutation relation $[\hat{q}_k, \hat{p}_k] = i\hbar$. Thus the nondissipative capacitance coupling circuit can be quantized. We use the following transformations [6] for the charge and current in the quantized Hamiltonian,

$$q_1{}' = \left(\frac{L_1}{L_2}\right)^{1/4} q_1 \cos\frac{\varphi}{2} - \left(\frac{L_2}{L_1}\right)^{1/4} q_2 \sin\frac{\varphi}{2},$$

$$q_2{}' = \left(\frac{L_1}{L_2}\right)^{1/4} q_1 \sin\frac{\varphi}{2} + \left(\frac{L_2}{L_1}\right)^{1/4} q_2 \cos\frac{\varphi}{2} \tag{3}$$

$$p_1{}' = \left(\frac{L_2}{L_1}\right)^{1/4} p_1 \cos\frac{\varphi}{2} - \left(\frac{L_1}{L_2}\right)^{1/4} p_2 \sin\frac{\varphi}{2},$$

$$p_1{}' = \left(\frac{L_2}{L_1}\right)^{1/4} p_1 \sin\frac{\varphi}{2} + \left(\frac{L_1}{L_2}\right)^{1/4} p_2 \cos\frac{\varphi}{2} \tag{4}$$

and take

$$\operatorname{tg}\varphi = \frac{2\sqrt{L_1L_2}}{L_2(1 + C/C_1) - L_1(1 + C/C_2)} \tag{5}$$

Then the quantized Hamiltonian of the system can be written as

$$\hat{H} = \frac{p_1{}'^2}{2\sqrt{L_1L_2}} + \frac{\alpha}{2} q_1{}'^2 + \frac{p_2{}'^2}{2\sqrt{L_1L_2}} + \frac{\beta}{2} q_2{}'^2 \tag{6}$$

where

$$\alpha = \sqrt{\frac{L_2}{L_1}}\left(\frac{1}{C} + \frac{1}{C_1}\right)\cos^2\frac{\varphi}{2} + \sqrt{\frac{L_1}{L_2}}\left(\frac{1}{C} + \frac{1}{C_2}\right)\sin^2\frac{\varphi}{2} + \frac{\sin\varphi}{C} \tag{7}$$

$$\beta = \sqrt{\frac{L_2}{L_1}}\left(\frac{1}{C} + \frac{1}{C_1}\right)\sin^2\frac{\varphi}{2} + \sqrt{\frac{L_1}{L_2}}\left(\frac{1}{C} + \frac{1}{C_2}\right)\cos^2\frac{\varphi}{2} - \frac{\sin\varphi}{C} \tag{8}$$

It is clear that Eq. (6) is the sum of the Hamiltonians of two independent quantum harmonic oscillators whose frequencies are, respectively,

$$\omega_1 = (\alpha/\sqrt{L_1L_2})^{1/2}, \qquad \omega_2 = (\beta/\sqrt{L_1L_2})^{1/2} \tag{9}$$

Thus we get the energies and eigenvectors of the system when $\varepsilon(t) = 0$,

$$E_{n_1,n_2} = (n_1 + 1/2)\hbar\omega_1 + (n_2 + 1/2)\hbar\omega_2 \tag{10}$$

$$|\psi_{n_1,n_2}\rangle = |n_1| \otimes |n_2\rangle, \qquad (n_1, n_2 = 0, 1, 2, \ldots) \tag{11}$$

where $|n_1\rangle$ and $|n_2\rangle$ represent the eigenvectors of oscillators with frequencies ω_1 and ω_2, respectively. We introduce the creation and annihilation operators for the two independent oscillators

$$a_k^+ = \left(\frac{\omega_k\sqrt{L_1L_2}}{2\hbar}\right)^{1/2}\left(q_k{}' - \frac{i}{\omega_k\sqrt{L_1L_2}}\, p_k{}'\right),$$
$$a_k = \left(\frac{\omega_k\sqrt{L_1L_2}}{2\hbar}\right)^{1/2}\left(q_k{}' + \frac{i}{\omega_k\sqrt{L_1L_2}}\, p_k{}'\right) \qquad (12)$$

From $[q_k, p_k] = i\hbar$, we have $[a_k, a_k^+] = 1$. Then we get

$$q_k{}' = \left(\frac{\hbar}{2\omega_k\sqrt{L_1L_2}}\right)^{1/2}(a_k^+ + a_k), \qquad p_k{}' = i\left(\frac{\hbar\omega_k\sqrt{L_1L_2}}{2}\right)^{1/2}(a_k^+ - a_k) \qquad (13)$$

For the commutation relations $[a_k, a_k^+] = 1$, we can define the vacuum state and Fock states. We assume the capacitance coupling circuit is in a displaced squeezed Fock state [7–9]:

$$|z_1, \xi_1, n_1; z_2, \xi_2, n_2\rangle = |z_1, \xi_1, n_1\rangle \otimes |z_2, \xi_2, n_2\rangle$$
$$= D(z_1)S(\xi_1)|n_1\rangle \otimes D(z_2)S(\xi_2)|n_2\rangle \qquad (14)$$

where $D(z_k)$ and $S(\xi_k)$ are the displacement operator and squeezing operator, respectively:

$$D(z_k) = \exp(z_k a_k^\dagger - z_k^* a_k), \qquad S(\xi_k) = \exp(\tfrac{1}{2}\xi_k a_k^{+2} - \tfrac{1}{2}\xi_k^* a_k^2) \qquad (15)$$

Here $z_k = |z_k|e^{i\theta_k}$ ($|z_k| > 0$, $0 \le \theta_k < 2\pi$) are the displacement parameters and $\xi_k = |\xi_k|e^{i\phi_k}$ ($|\xi_k| > 0$, $0 \le \phi_k < 2\pi$) the squeezing parameters. Using the formula

$$e^{\lambda A}Be^{-\lambda A} = \hat{B} + \lambda[\hat{A}, \hat{B}] + \frac{\lambda^2}{2!}[\hat{A}, [\hat{A}, \hat{B}]] + \cdots \qquad (16)$$

we can easily prove the following relations:

$$D^+(z_k)a_kD(z_k) = a_k + z_k, \qquad D^+(z_k)a_k^+D(z_k) = a_k^+ + z_k^* \qquad (17)$$
$$S^+(\xi_k)a_kS(\xi_k) = a_k\cosh|\xi_k| + a_k^+e^{i\phi_k}\sinh|\xi_k| \qquad (18)$$
$$S^+(\xi_k)a_k^+S(\xi_k) = a_k^+\cosh|\xi_k| + a_ke^{-i\phi_k}\sinh|\xi_k| \qquad (19)$$

From Eqs. (13) and (17)–(19), we obtain

$$S^{+}(\xi_k)\, D^{+}(z_k) q_k'\, D(z_k) S(\xi_k) = \left(\frac{\hbar}{2\omega_k \sqrt{L_1 L_2}}\right)^{1/2} [A(\xi_k) a_k + A^*(\xi_k) a_k^{+} + B(z_k)] \tag{20}$$

$$S^{+}(\xi_k)\, D^{+}(z_k) p_k'\, D(z_k) S(\xi_k) = i\left(\frac{\hbar\omega_k \sqrt{L_1 L_2}}{2}\right)^{1/2} [E(\xi_k) a_k - E^*(\xi_k) a_k^{+} + F(z_k)] \tag{21}$$

where

$$A(\xi_k) = \cosh|\xi_k| + e^{-i\phi_k} \sinh|\xi_k|, \qquad B(z_k) = 2|z_k| \cos\theta_k \tag{22}$$

$$E(\xi_k) = e^{-i\phi_k} \sinh|\xi_k| - \cosh|\xi_k|, \qquad F(z_k) = -2i|z_k| \sin\theta_k \tag{23}$$

From Eqs. (20) and (21) and the formula $e^{\lambda A} B^m e^{-\lambda A} = (e^{\lambda A}\, B e^{-\lambda A})^m$, we get

$$\begin{aligned} S^{+}(\xi_k)\, D^{+}(z_k) q_k'^2\, D(z_k) S(\xi_k) &= \frac{\hbar}{2} \frac{1}{\omega_k \sqrt{L_1 L_2}} [|A(\xi_k)|^2 (2a_k^{+} a_k + 1) \\ &\quad + B^2(z_k) + A^2(\xi_k) a_k^2 + A^{*2}(\xi_k) a_k^{+2} \\ &\quad + 2B(z_k) A(\xi_k) a_k + 2B(z_k) A^*(\xi_k) a_k^{+}] \end{aligned} \tag{24}$$

$$\begin{aligned} S^{+}(\xi_k)\, D^{+}(z_k) p_k'^2\, D(z_k) S(\xi_k) &= \frac{\hbar}{2} \omega_k \sqrt{L_1 L_2} [|E(\xi_k)|^2 (2a_k^{+} a_k + 1) \\ &\quad - F^2(z_k) - E^2(\xi_k) a_k^2 - E^{*2}(\xi_k) a_k^{+2} \\ &\quad - 2F(z_k) E(\xi_k) a_k + 2F(z_k) E^*(\xi_k) a_k^{+}] \end{aligned} \tag{25}$$

Therefore, the mean values and mean-square values of q_k' and p_k' in the state given by Eq. (14) can be obtained, respectively, as

$$\langle q_k' \rangle = \left(\frac{\hbar}{2\omega_k \sqrt{L_1 L_2}}\right)^{1/2} B(z_k), \qquad \langle p_k' \rangle = i\left(\frac{\hbar\omega_k \sqrt{L_1 L_2}}{2}\right)^{1/2} F(z_k) \tag{26}$$

$$\langle q_k'^2 \rangle = \frac{\hbar}{2} \frac{1}{\omega_k \sqrt{L_1 L_2}} [|A(\xi_k)|^2 (2n_k + 1) + B^2(Z_k)] \tag{27}$$

$$\langle p_k'^2 \rangle = \frac{\hbar}{2} \omega_k \sqrt{L_1 L_2} [|E(\xi_k)|^2 (2n_k + 1) - F^2(Z_k)] \tag{28}$$

Then, using Eqs. (3), (4) and (26)–(28), we get the mean values and mean-square values of the charge and current of the capacitance coupling circuit in the state given by Eq. (14). They are, respectively,

$$\langle q_1 \rangle = \left(\frac{\hbar}{2L_1}\right)^{1/2}\left[\frac{1}{\sqrt{\omega_1}} B(z_1) \cos\frac{\varphi}{2} + \frac{1}{\sqrt{\omega_2}} B(z_2) \sin\frac{\varphi}{2}\right] \tag{29}$$

$$\langle q_2 \rangle = \left(\frac{\hbar}{2L_2}\right)^{1/2}\left[\frac{-1}{\sqrt{\omega_1}} B(z_1) \sin\frac{\varphi}{2} + \frac{1}{\sqrt{\omega_2}} B(z_2) \cos\frac{\varphi}{2}\right] \tag{30}$$

$$\langle p_1 \rangle = i\left(\frac{\hbar L_1}{2}\right)^{1/2}\left[\sqrt{\omega_1} F(z_1) \cos\frac{\varphi}{2} + \sqrt{\omega_2} F(z_2) \sin\frac{\varphi}{2}\right] \tag{31}$$

$$\langle p_2 \rangle = i\left(\frac{\hbar L_2}{2}\right)^{1/2}\left[-\sqrt{\omega_1} F(z_1) \sin\frac{\varphi}{2} + \sqrt{\omega_2} F(z_2) \cos\frac{\varphi}{2}\right] \tag{32}$$

$$\begin{aligned}\langle q_1{}^2 \rangle = \frac{\hbar}{2}\frac{1}{L_1}\Bigg\{&\frac{1}{\omega_1}[|A(\xi_1)|^2(2n_1+1) + B^2(z_1)]\cos^2\frac{\varphi}{2} \\ &+ \frac{1}{\omega_2}[A(\xi_2)|^2(2n_2+1) + B^2(z_2)]\sin^2\frac{\varphi}{2} \\ &+ \frac{1}{\sqrt{\omega_1\omega_2}} B(z_1)B(z_2)\sin\varphi\Bigg\}\end{aligned} \tag{33}$$

$$\begin{aligned}\langle q_2{}^2 \rangle = \frac{\hbar}{2}\frac{1}{L_2}\Bigg\{&\frac{1}{\omega_1}[A(\xi_1)|^2(2n_1+1) + B^2(z_1)]\sin^2\frac{\varphi}{2} \\ &+ \frac{1}{\omega_2}[A(\xi_2)|^2(2n_2+1) + B^2(z_2)]\cos^2\frac{\varphi}{2} \\ &- \frac{1}{\sqrt{\omega_1\omega_2}} B(z_1)B(z_2)\sin\varphi\Bigg\}\end{aligned} \tag{34}$$

$$\begin{aligned}\langle p_1{}^2 \rangle = \frac{\hbar}{2} L_1\{&\omega_1[|E(\xi_1)|^2(2n_1+1) - F^2(z_1)]\cos^2\frac{\varphi}{2} \\ &+ \omega_2[|E(\xi_2)|^2(2n_2+1) - F^2(z_2)]\sin^2\frac{\varphi}{2} \\ &- \sqrt{\omega_1\omega_2} F(z_1)F(z_2)\sin\varphi\}\end{aligned} \tag{35}$$

$$\langle p_1{}^2 \rangle = \frac{\hbar}{2} L_2\{\omega_1[|E(\xi_1)|^2(2n_1+1) - F^2(z_1)]\sin^2\frac{\varphi}{2}$$

$$+ \omega_2[|E(\xi_2)|^2(2n_2 + 1) - F^2(z_2)] \cos^2\frac{\varphi}{2} + \sqrt{\omega_1\omega_2}F(z_1)F(z_2) \sin\varphi\} \tag{36}$$

From the above equations, we can see that the mean values for both the charge and current only depend on the displacement parameters z_k, while the mean-square values are dependent on both the displacement parameters z_k and squeezing parameters ξ_k. Then the quantum fluctuations of charge and current for the capacitance coupling circuit in the displaced squeezed Fock state are

$$\langle(\Delta q_1)^2\rangle = \frac{\hbar}{2}\frac{1}{L_1}\left[\frac{1}{\omega_1}|A(\xi_1)|^2(2n_1 + 1)\cos^2\frac{\varphi}{2} + \frac{1}{\omega_2}|A(\xi_2)|^2(2n_2 + 1)\sin^2\frac{\varphi}{2}\right] \tag{37}$$

$$\langle(\Delta q_2)^2\rangle = \frac{\hbar}{2}\frac{1}{L_2}\left[\frac{1}{\omega_1}|A(\xi_1)|^2(2n_1 + 1)\sin^2\frac{\varphi}{2} + \frac{1}{\omega_2}|A(\xi_2)|^2(2n_2 + 1)\cos^2\frac{\varphi}{2}\right] \tag{38}$$

$$\langle(\Delta p_1)^2\rangle = \frac{\hbar}{2}L_1\left[\omega_1|E(\xi_1)|^2(2n_1 + 1)\cos^2\frac{\varphi}{2} + \omega_2|E(\xi_2)|^2(2n_2 + 1)\sin^2\frac{\varphi}{2}\right] \tag{39}$$

$$\langle(\Delta p_2)^2\rangle = \frac{\hbar}{2}L_2\left[\omega_1|E(\xi_1)|^2(2n_1 + 1)\sin^2\frac{\varphi}{2} + \omega_2|E(\xi_2)|^2(2n_2 + 1)\cos^2\frac{\varphi}{2}\right] \tag{40}$$

where φ, ω_k, $A(\xi_k)$, and $E(\xi_k)$ are given by Eqs. (5), (9), (22), and (23), respectively. From the results we find that there exist quantum fluctuations for both charge and current in each circuit, and the fluctuations do not depend on the displacement parameters z_k. The magnitude of the fluctuations is not only related to the device and the squeezed parameters ξ_k, but is also associated with the other circuit and the magnitude of the coupling capacitance. Therefore, the quantum fluctuations of the charge and current in each component circuit are interrelated. In particular, when n_k and the parameters z_k and ξ_k take special values, the quantum fluctuations corresponding to special states can be obtained. For example, when $z_k = 0$ and $n_k = 0$, the quantum fluctuations of charge and current in the squeezed vacuum state are, respectively,

$$\langle(\Delta q_1)^2\rangle = \frac{\hbar}{2}\frac{1}{L_1}\left[\frac{1}{\omega_1}|A(\xi_1)|^2\cos^2\frac{\varphi}{2} + \frac{1}{\omega_2}|A(\xi_2)|^2\sin^2\frac{\varphi}{2}\right] \tag{41}$$

$$\langle(\Delta q_2)^2\rangle = \frac{\hbar}{2}\frac{1}{L_2}\left[\frac{1}{\omega_1}|A(\xi_1)|^2\sin^2\frac{\varphi}{2} + \frac{1}{\omega_2}|A(\xi_2)|^2\cos^2\frac{\varphi}{2}\right] \tag{42}$$

$$\langle(\Delta p_1)^2\rangle = \frac{\hbar}{2}L_1\left[\omega_1|E(\xi_1)|^2\cos^2\frac{\varphi}{2} + \omega_2|E(\xi_2)|^2\sin^2\frac{\varphi}{2}\right] \tag{43}$$

$$\langle(\Delta p_2)^2\rangle = \frac{\hbar}{2}L_2\left[\omega_1|E(\xi_1)|^2\sin^2\frac{\varphi}{2} + \omega_2|E(\xi_2)|^2\cos^2\frac{\varphi}{2}\right] \tag{44}$$

In the same way, from Eqs. (37)–(40), the quantum fluctuations of charge and current are obtained easily, respectively, in the squeezed Fock state ($z_k = 0$), squeezed state ($n_k = 0$), displaced Fock state ($\xi_k = 0$), coherent state ($\xi_k = 0$ and $n_k = 0$), Fock state ($z_k = 0$ and $\xi_k = 0$), and vacuum state ($z_k = 0$, $\xi_k = 0$, and $n_k = 0$). These special states can be regarded as special cases of the displaced squeezed Fock state when n_k and the parameters z_k and ξ_k take corresponding special values. Therefore, it is more general and significant to study the quantum fluctuations of charge and current in displaced squeezed Fock states.

3. CONCLUSION

In this paper, on the basis of the equations of motion for a capacitance coupling circuit, we studied the quantum fluctuations of the charge and current of a nondissipative mesoscopic capacitance coupling circuit in a displaced squeezed Fock state. The quantum fluctuations are not only related to the device and the squeezed parameters, but also to the other circuit and the magnitude of the coupling capacitance. As the displaced squeezed Fock state can be reduced to the vacuum state and squeezed vacuum state, the previously discussed fluctuations of the mesoscopic capacitance coupling circuit are special cases of our general results. Therefore, it is more general and significant to study the quantum fluctuations of charge and current in displaced squeezed Fock states.

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